Faulty cellular automata: reliable computations in finite and infinite time.

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Faulty CA:
Let A be a deterministic CA. A PCA $\mathcal{A}$ is called an ϵ-perturbation of A if

$$\mathcal{A}(\cdot) = \begin{cases} A(\cdot), & \text{with prob } \geq 1-\epsilon, \\ \text{whatever}, & \text{with prob } \leq \epsilon. \end{cases}$$

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- his *fault* is a failure of an update
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- Do you have problems that can be solved with these techniques?
- Do you have new techniques to solve these open problems?

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A weak property: There exist two initial configuration $\mathcal{I}_0$ and $\mathcal{I}_1$ and $\delta > 0$ such that for all steps t (a finite version: for $t < exp\{n\}$)

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A somewhat stronger property: There exist two different $\mathcal{A}_\epsilon$-invariant measures μ_0 and μ_1.

Toom theorem: There exists a deterministic CA on $\{0,1\}^{\mathbb{Z}^2}$ such that small enough ϵ, for all ϵ-perturbations $\mathcal{A}$ of this automaton *a natural property* holds: if we start with the initial configuration $\mathcal{I}_0$ (all zeros) or $\mathcal{I}_1$ (all ones), then for each step t, for each cell (i,j)

$$\mathrm{Prob}[(\mathcal{A}^t(\mathcal{I}_0))_{ij} = 0] > 1 - O(\varepsilon),$$

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A weaker corollary: There exist two initial configuration $\mathcal{I}_0$ (all zeros) and $\mathcal{I}_1$ (all ones) and $\delta > 0$ such that for all steps t

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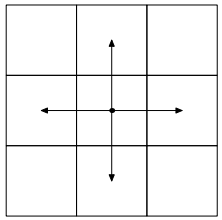
A somewhat stronger corollary: There exist two different $\mathcal{A}_\epsilon$-invariant measures μ_0 (most cells are 0) and μ_1 (most cells are 1).

Isn't **the Toom theorem** trivial? Let's take the majority vote

$$A(i,j,t+1) = \mathrm{maj}\{A(i,j,t), A(i+1,j,t), A(i-1,j,t), A(i,j+1,t), A(i,j-1,t)\}$$



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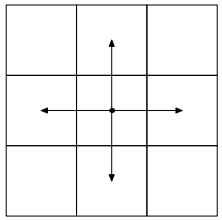

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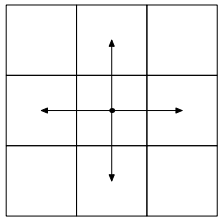

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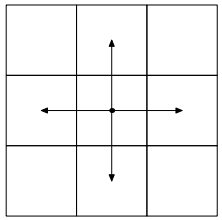

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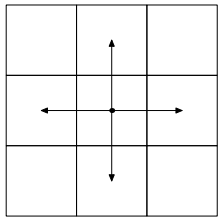

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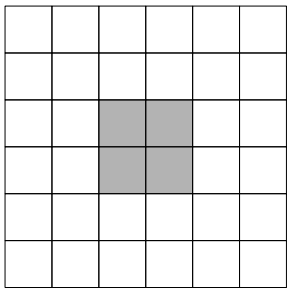
What is wrong with the symmetric *majority* rule? It does not erode an island of 1-s in the ocean of 0-s.



The idea of Andrei Toom: take **NEC** majority:

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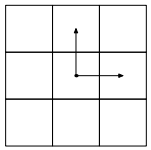

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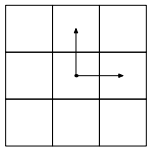

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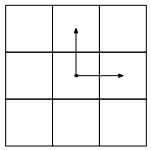

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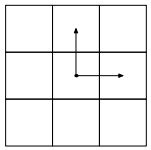
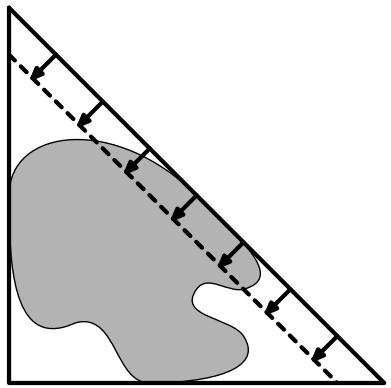


What so good about **NEC** majority?
Every island of 1-s in the ocean of 0-s shrinks. Moreover, it shrinks in *fast*. (Remember the slides of Lise Ponselet!)



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- small islands with a small clean "security zones" around,
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Step 2 (combinatorial):

- small islands shrink without interacting with other islands,
- larger islands shrink without interacting with other islands,
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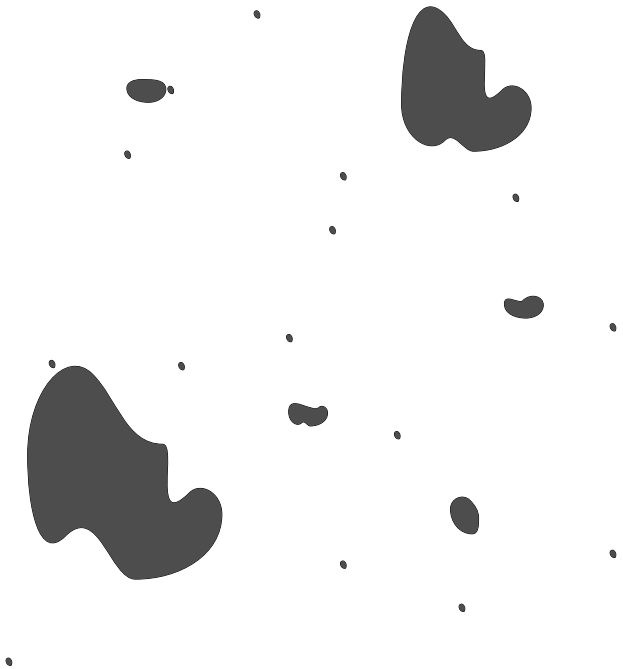
Metaphorical picture of **Step 1**: split faults into isolated islands



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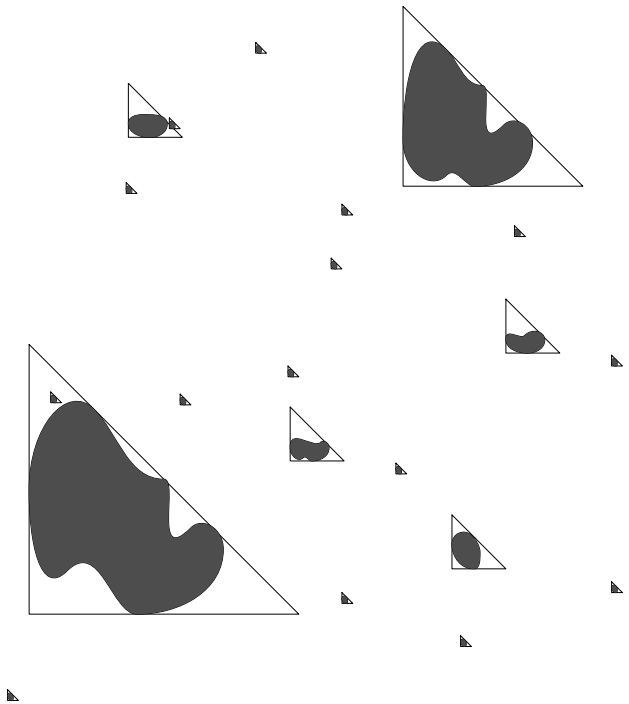

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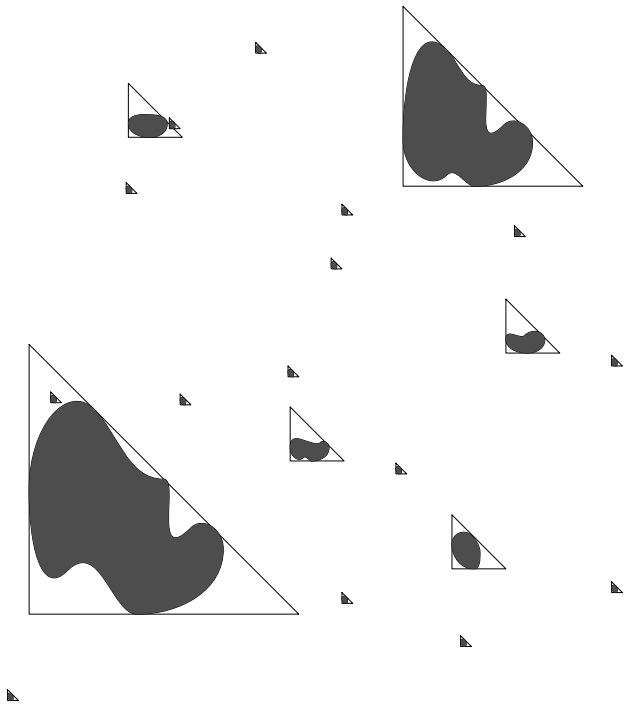

Conclusion: Only small neighborhoods of islands are affected by the random faults. Information about the initial state is kept forever.

Extensions of this technique:

- Finite case (torus $\mathbb{Z}_n^2$): The initial information is kept until a too large islands of errors comes.
- More accurate bounds for the finite case.
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- Other invariant measures for an ϵ-perturbation of the NEC majority?

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Remark 1: The fine version of the construction also works fine.
Remark 2: Essentially we *repeat* the argument from the Toom theorem. We cannot (?) use Toom's theorem as a black box.

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- we simulate 1D computation in a 3D medium.

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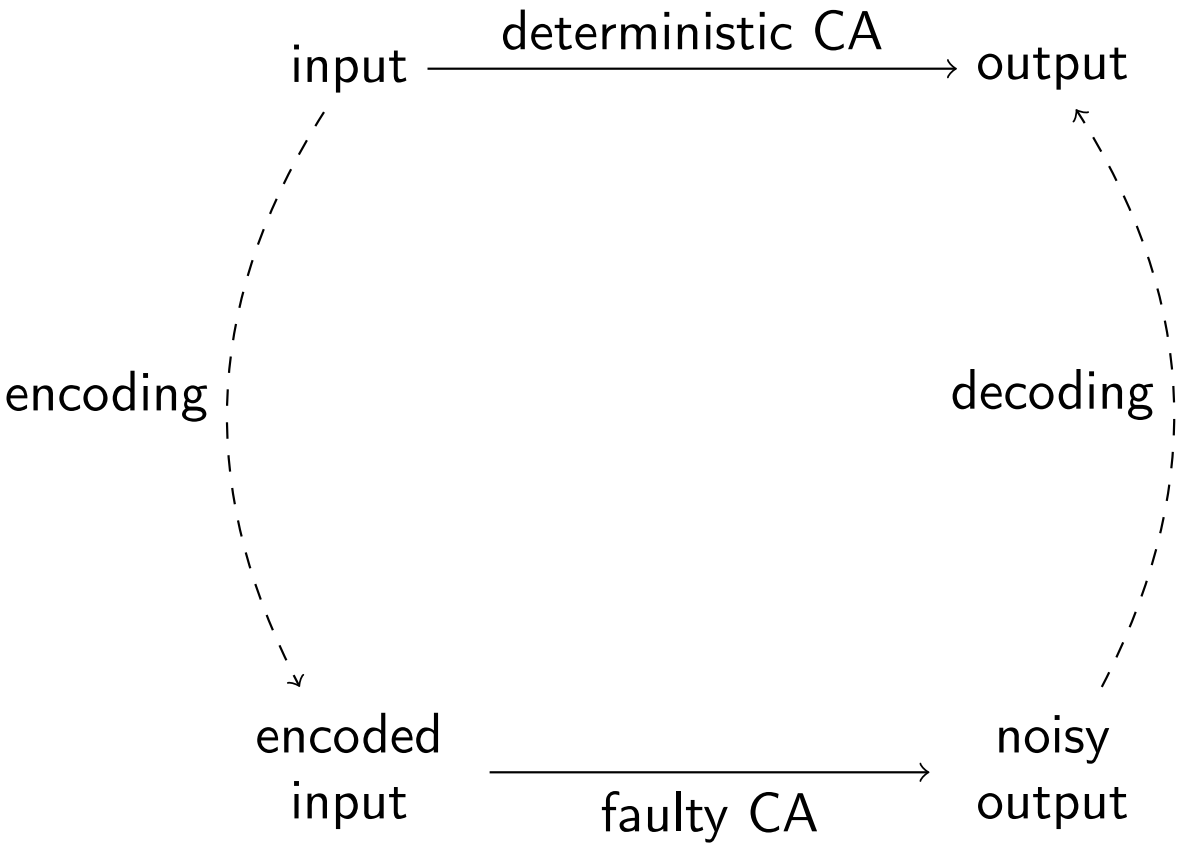
Under the hood of Gács' construction:

- Each cell "knows" its coordinates in the block and "counts" its steps in the loop. Self-correction *á la* Toom maintains the correct information about coordinates and the counter.
- Self-referential constriction *á la* Kleene provides self-simulation.
- Simple duplication helps to suppress small islands of errors.

Pitfalls of "robust simulation"



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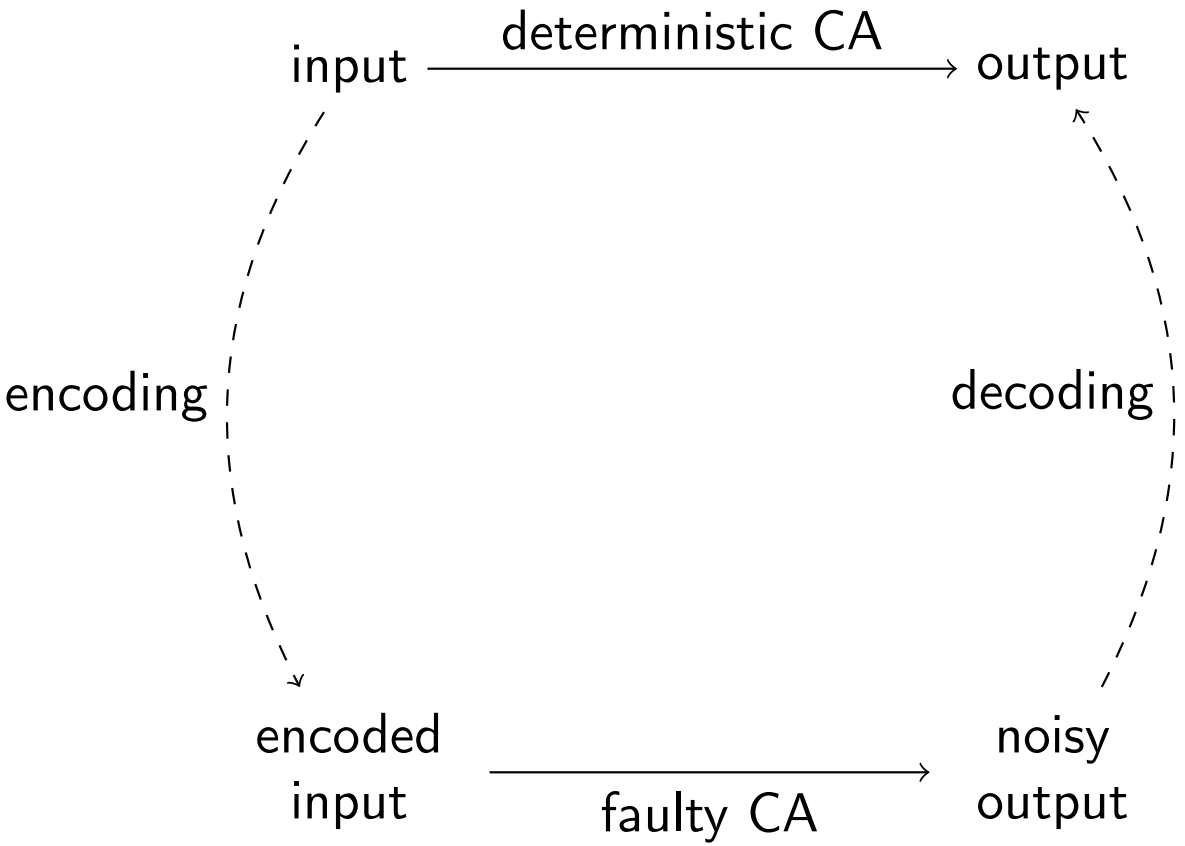

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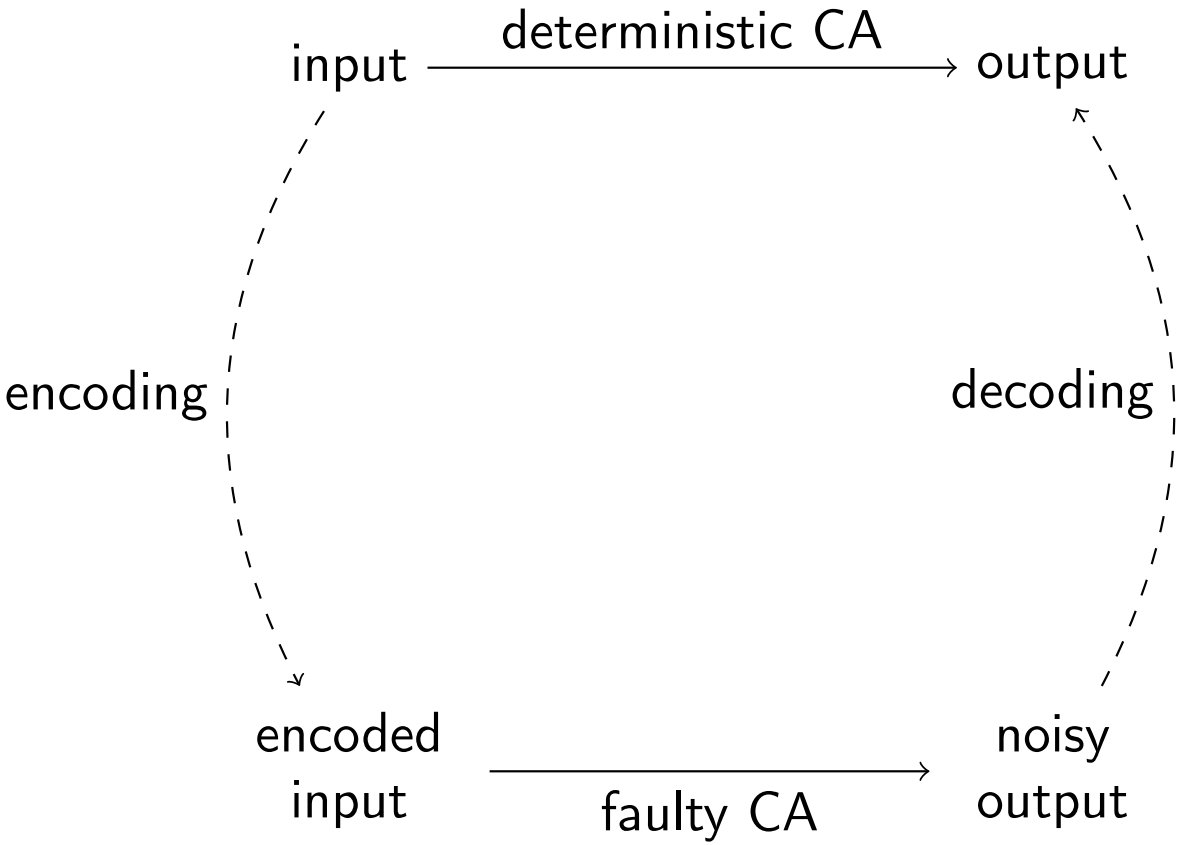

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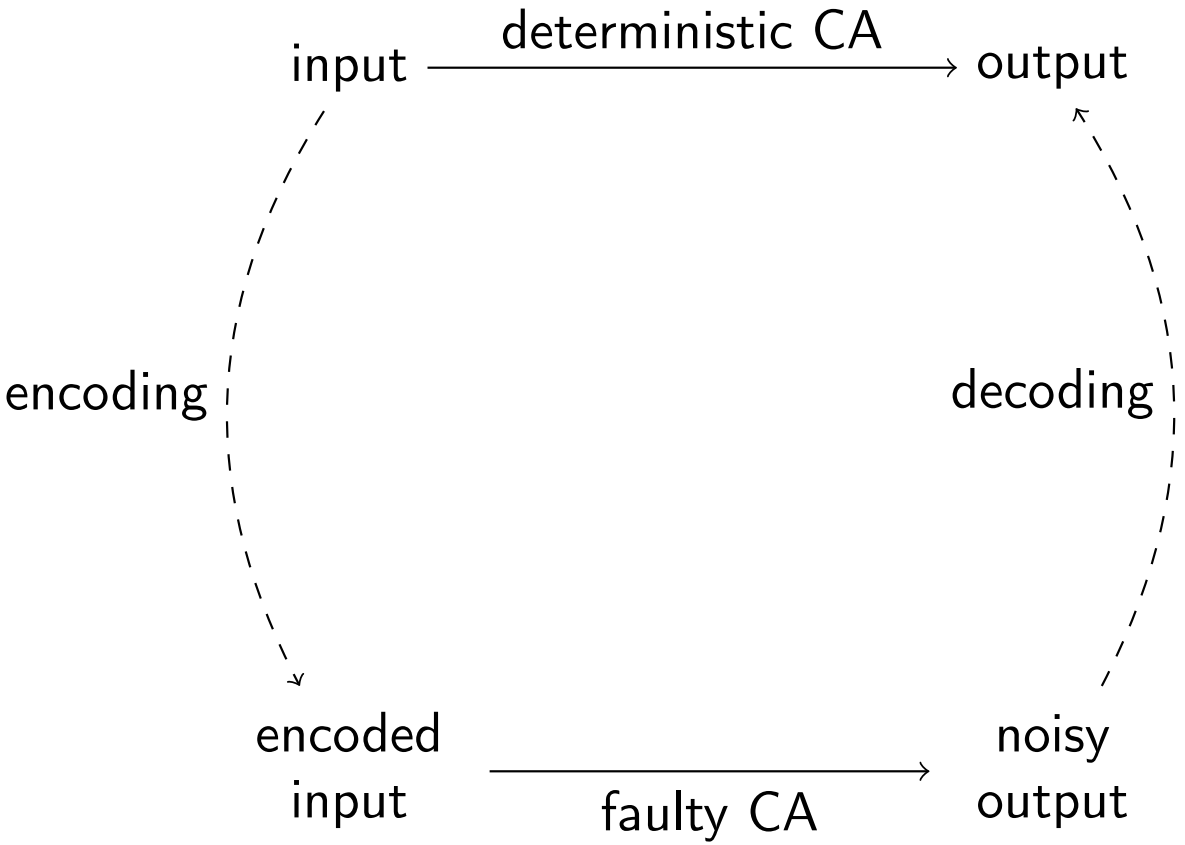

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Open problem: How to define robust simulation for an infinite medium?

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Beyond this talk: an automata *á la* Gács can keep information with positive density ($O(1)$ bits/cell).

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Several questions remain:

- A simpler construction of a nonergodic faulty CA in 1D.
- Reliable computations in infinite medium: find the "right" definition.
- "Smooth" universality: can a faulty CA be intrinsically universal for PCA's in any reasonable sense? [*cf.* Arrighi-Schabanel-Theyssier'12]

Merci de votre attention !